

Introduction

This Chapter describes the Antecedent Precipitation Index (API) runoff Operation (API-SLC) developed by the Colorado Basin River Forecast Center (CBRFC) River Forecast Center.

API procedures were first defined in the 1940's by M. A. Kohler (Reference 1). During this period of hydrology, scientists were seeking techniques which would simplify the relationships of rainfall and runoff. Various techniques which tried to conceptualize soil characteristics, through the application of infiltration theory and other models, were too complex especially when trying to apply them to a very large basin. A more important consideration in forecasting is the time required to produce the product. Without computers alternate less time-consuming methods were needed (References 5 and 6).

Availability of input parameters was another consideration in model selection. Generally storm characteristics can be determined from an adequate network of precipitation stations but determining soil moisture conditions throughout the basin is difficult. Variations in soil and surface characteristics, vegetation differences and land use add to the complexity. Many factors have been used to index the moisture conditions such as:

- o days since last rain
- o discharge at the beginning of the storm
- o antecedent precipitation

The first index is obviously insensitive because it only accounts for the duration of the drought and does not take into effect recharge to the basin. The second is seasonally sensitive and does not reflect changes by previous rains. Antecedent precipitation generally provides good results, provided it is properly derived and uses a seasonal index or temperature.

The variable API, for which the procedure is named, is a rough representation of the initial soil-moisture condition and can also be easily determined. It tries to utilize the accumulated precipitation and, at the same time, take into account evaporation and infiltration.

By using API, week of the year and storm precipitation and duration as parameters, Kohler and Linsley (Reference 2) developed a relationship between storm runoff and precipitation by a graphical method of coaxial relations. It is based on the premise that if any important factor is omitted from a relation, then the scatter of points in a plotting of observed values of the dependent variable versus those computed by the relation will be at least partially explained. The API procedure is really a set of three-variable relations arranged with common axes to facilitate computation.

The Colorado Basin River Forecast Center (CBRFC) has adapted these procedures and applied them to basins in Arizona. Modifications were

made to alter the lower limit of the API index, adjust the API recession based on simulated percent areal snow cover and allow duration to be affected by differing 6 hour significant precipitation levels.

API Model (Reference 3)

The API model consists of 3 three-variable relations (Figure 1), relating basin recharge as the dependent variable to the antecedent precipitation (API), date (week number), the rainfall amount and the rainfall duration as the independent variables. Basin recharge is defined as the loss due to interception, infiltration and depression storage or basically the difference between precipitation and runoff.

Chart A in Figure 1 is the API versus basin recharge, with the points labeled with the week numbers. A family of curves is fitted to the points with one curve for each week. Chart B is the observed recharge versus the computed recharge with the points labeled for rainfall storm duration (hours). Again a family of curves is drawn defining the effect of duration on recharge. Chart C is observed basin recharge versus computed recharge with the points labeled with rainfall amounts. Chart D displays the accuracy of the procedure of the other three charts. It is a plot of observed recharge versus computed recharge.

The calibration process is successive approximations of curve selection to converge to the best graphical solution. The methods for adjusting the relationships are made by alternating the entrance into the procedure from Chart A through D and then D through A.

The API as used in the CBRFC model (Reference 4) is slightly modified to facilitate its usage in computer applications (Figure 2). The precipitation curves have been swapped with duration curves. The duration quadrant has re-introduced the antecedent precipitation and season indices as a parameter for effective duration. Also the output has been changed to display runoff directly reducing the need to subtract basin recharge from precipitation.

First Quadrant (Season) (Reference 4)

The first quadrant is a relation of API versus basin recharge. The points are labeled with the week number and a family of curves drawn to represent the date or seasonal effect on basin recharge. The following equation defines these curves:

$$RI1 = (A + B*Y) * (C)**API$$

$$B = (I-A)/2$$

$$G1 = (E2 - E1)/2$$

$$G2 = (E1 - E2)/2$$

For weeks between WN and WX:

$$Y = 1 - (\text{COS}((W-WN) (\text{Pi}/(WX-WN))))**CP$$

$$C = E1 + G1 * ((W - WN) / (WX - WN)) / 2)$$

For weeks between WX and 52:

$$Y = 1 + (\text{COS}((W - WX) (\text{Pi} / (52 + WN - WX)))) ** CP$$

$$C = E2 + G2 * ((W - WX) / (52 + WN - WX)) / 2)$$

For weeks between 52 and WN:

$$Y = 1 + (\text{COS}(W + 52 - WX) (\text{Pi} / (52 + WN - WX)))) ** CP$$

$$C = E2 + G2 * ((52 + W - WX) / (52 + WN - WX)) / 2)$$

where A is the intercept of WN in the RI axis
 I is the intercept of WX in the RI axis
 WN is the wettest week number
 WX is the driest week number
 W is the week number of the current storm
 E1 is the curvature constant for WN
 E2 is the curvature constant for WX
 G1 determines the rate at which E1 approaches E2
 G2 determines the rate at which E2 approaches E1
 CP determines the distribution of week curves
 API is the Antecedent Precipitation Index

A CP value of 1.0 distributes the week curves evenly between WX and WN. As CP approaches zero, the week curves tend to pack around WX and WN. When CP increases above 1, the week curves cluster midway between WX and WN. See Figures 3 and 4 for relationship of parameters.

The antecedent precipitation index is generally defined by the equation:

$$API = b_1 P_1 + b_2 P_2 + b_3 P_3 + \dots + b_i P_i$$

where P_i is the amount of precipitation i day prior to storm
 b_i is a constant as function of time $1/i$

For this model the decrease with time has been assumed to follow a logarithmic decay rather than a reciprocal. Thus during periods of no precipitation:

$$API_i = k * API_{i-1}$$

For periods with precipitation:

$$API_i = (API_{i-1} + \text{Precip}) * k$$

The API index for any day is equal to that of the previous day multiplied by the factor K. If any rain occurs it is added to the index (Figure 5). The value k varies with physiographic basin characteristics, evaporation, temperature and humidity. However through the use of other factors, such as the week or seasonal term, most variation has been accounted. Through experimentation (Reference 3), k is important, though not critical and ranges in value from 0.85 to 0.90 over most of the eastern and central portions of the United States.

The antecedent precipitation index is computed from mean areal precipitation (MAP) provided as output from the snow accumulation and ablation model. In areas of snowfall, the precipitation is applied to the model on days it melts rather than when it falls. This prevents over forecasting of events by applying water-equivalent of the snow at occurrence.

Snow cover provides for a modification to the API calculation. As percent areal snow cover approaches 100 percent, moisture loss is reduced. The term k is modified by a snow term to reduce the API reduction from day to day:

$$API_1 = API_0 - API_0 * (1.-k)*(1.-\text{fraction of snow cover})$$

For areas in Arizona with long periods of drought, API is allowed to decrease below zero to minimum of -0.99 inches. If the lower limit of API is selected to be negative, the method of reducing API changes. Once API reaches .05 inches, a constant increment of .01 inches per day is subtracted until the API value reaches the lower limit. The value of API used in the model is set at .01 until API reaches a positive value. This process simulates an increased soil moisture capacity which must be satisfied before API is allowed to increase.

Second Quadrant (Storm Precipitation) (Reference 4)

This quadrant gives the relation of observed basin recharge versus computed recharge. Points are labeled with precipitation in inches and a family of curves of storm totals is drawn. This represents the effect of precipitation on recharge under the conditions calculated in the first quadrant. The following equation defines this curve:

$$RI2 = P * (P/(P+1))^{**}RI1$$

where P is observed precipitation

Precipitation is obtained from mean areal precipitation calculated directly from one of the precipitation models. The MAP could be modified by the snow accumulation and ablation model before being used as input to the API model.

Third Quadrant (Storm Duration) (Reference 4)

This relation is observed basin recharge versus runoff with points labeled on the basis of storm duration. Basin recharge as explained through the first quadrant relationship is re-introduced as a parameter of effective duration. The equation of the curves is defined as follows:

$$RO = RI2 * (K)^{**}FD$$

$$FD = (DUR*(RI1 + 1))/(6 + M * (RI1)^{**}POW$$

where DUR is storm duration in hours

M, POW and K are constants (K is less than 1)

Average duration over the entire basin is difficult to determine but it is not critical when limited to 6 hourly rainfall data. This model keeps track of duration based on 6 hourly significant rainfall. Results from experimental infiltration data show a value of 0.10 inches is a good default for significant rainfall level but the significant level can be altered to take into account variations across the United States.

Runoff

Runoff is produced through a series of equations approximating the curves of the API coaxial graphical method. This runoff represents surface flow for a specific period. For this operation, the period is fixed at 24 hours in the 12Z to 12Z time frame.

In order to provide an output, runoff time series with a 6 hour data time interval, 24 hour runoff is distributed in the same percentage as precipitation for the same 24 hour period. Thus it is assumed that each 6 hour period of precipitation is an antecedent precipitation index.

An alternative to this method would be computing runoff depths from accumulated precipitation up to the end of a 6 hour period and subtracting successive values of runoff.

The relative accuracies of the two techniques are dependent upon the adequacy of the assumed weights for antecedent precipitation. The first method is preferred because it gives more significance to time variations of rainfall intensity and may, therefore, provide for more accurate computations.

Conclusions

The effect amount and distribution of antecedent precipitation has upon storm runoff depends upon the extent to which it has been dissipated through evaporation, transpiration, etc. Through the API coaxial relationships, a generally high correlating procedure can provide a simple method of computing runoff.

There are some limitations which directly effect reliability or use of such models. Most problems can be overcome utilizing input from the professional hydrologist. The following difficulties are considered to be some of the major deficiencies:

- o a relation based on storms of uniform areal distribution will yield runoff values which are too low when applied to storms of extremely uneven distribution
- o rainfall intensity is omitted or is generally smoothed into 6 hour periods
- o the procedure does not model frozen ground

References

1. M. A. Kohler, 'The Use of Crest Stage Relations in Forecasting the Rise and Fall of the Flood Hydrograph', U.S. Weather Bureau, 1944 (mimeo).
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3. M. A. Kohler and R. K. Linsley, 'Predicting the Runoff from Storm Rainfall', U.S. Weather Bureau Research Paper No. 34, 1951.
4. J. P. McCallister, 'Role of Digital Computers in Hydrological Forecasting and Analyses', U. S. Geological Survey Publication No. 63, 1963.
5. M. A. Kohler and M. M. Richards, 'Multicapacity Basin Accounting for Predicting Runoff from Storm Precipitation', Journal of Geophysical Research, Dec. 1962.
6. R. P. Betson, R. L. Tucker and F. M. Haller, 'Using Analytical Methods to Develop a Surface-Runoff Model', Water Resources Research, Feb. 1969.

Figure 1. Coaxial relationship - Antecedent Precipitation Index
 (Chart A, Chart B, Chart C and Chart D)

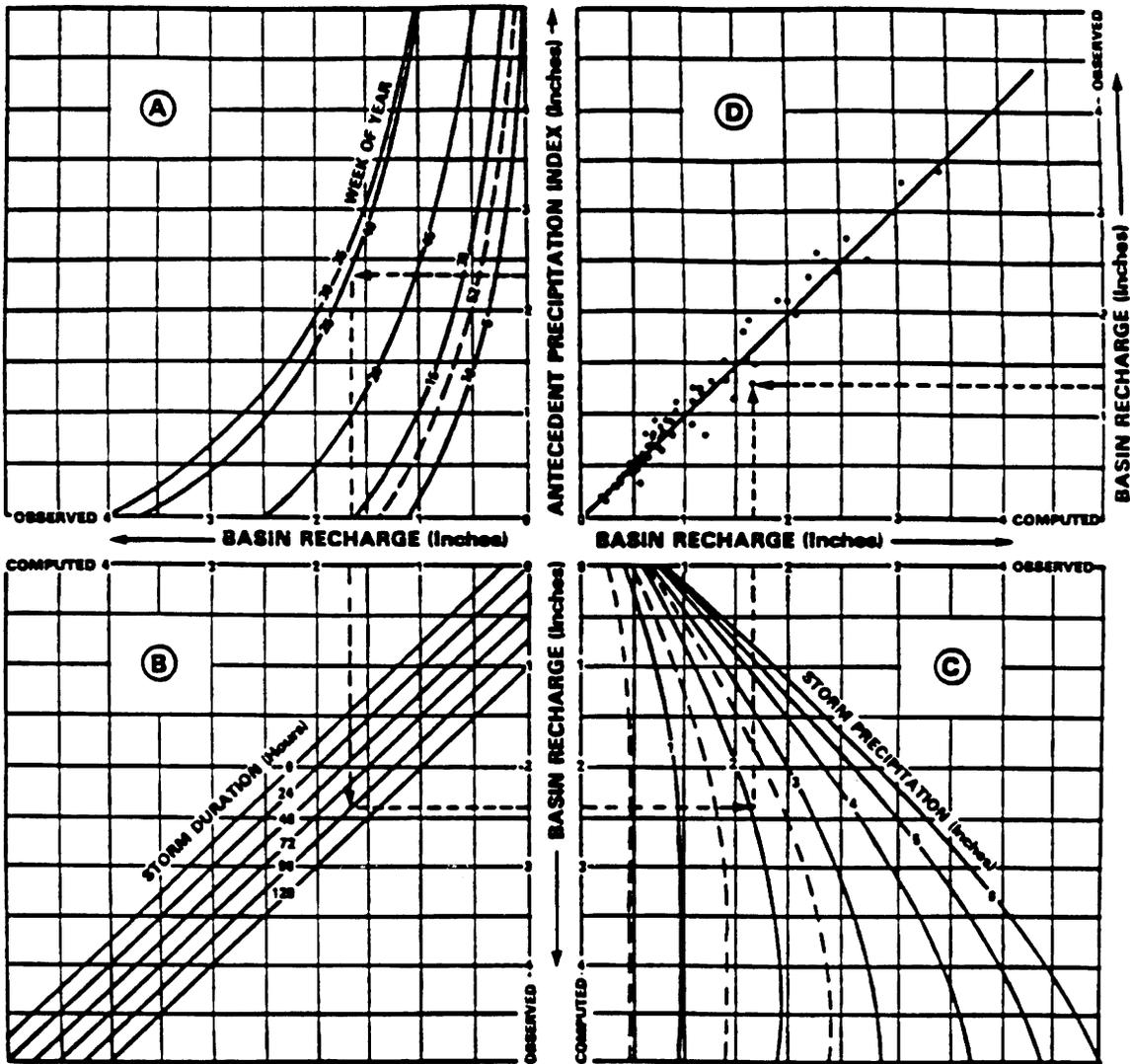


Figure 1. Coaxial relation -- Antecedent Precipitation Index

Figure 2. Modified API - Salt Lake City RFC

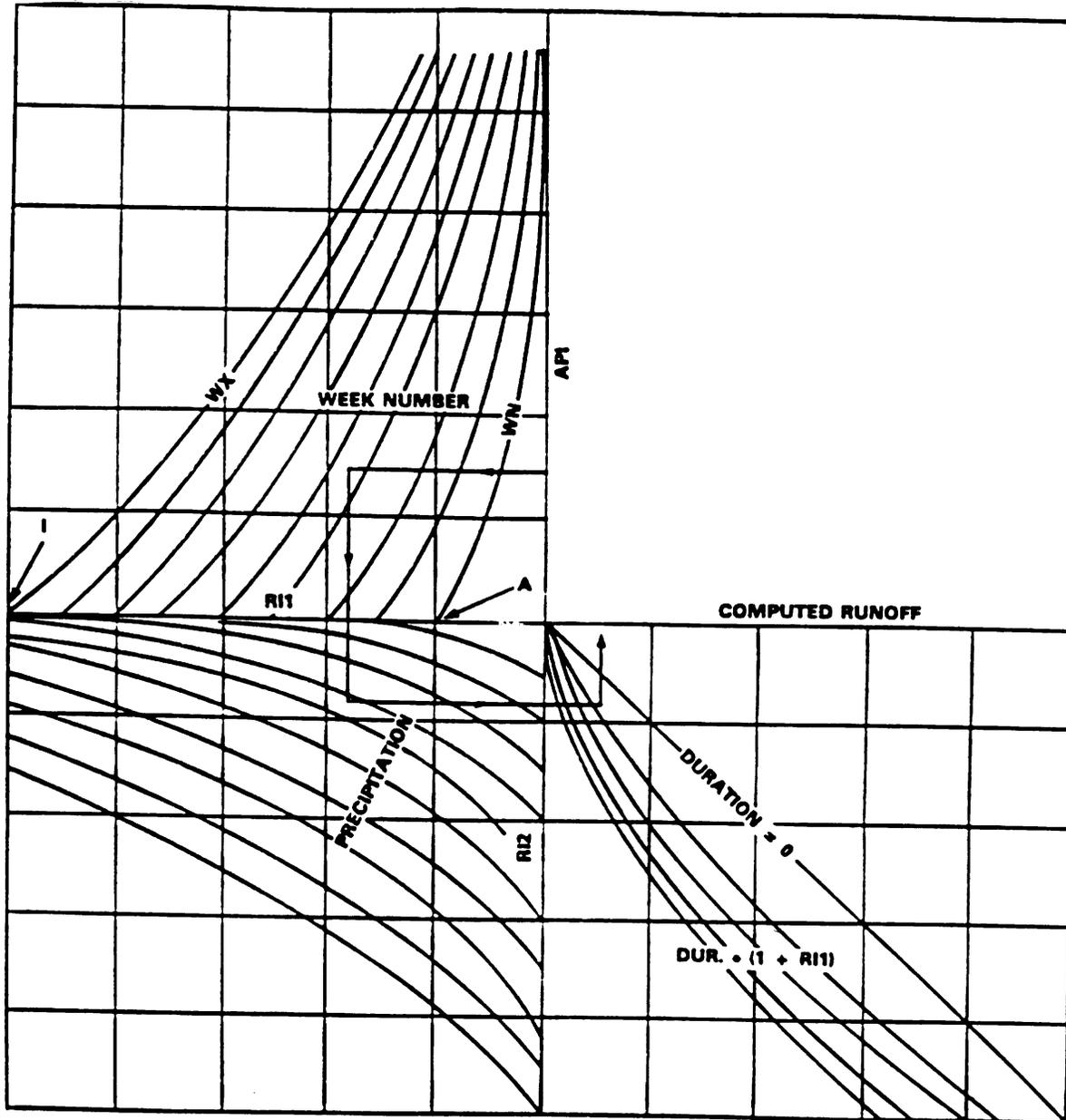
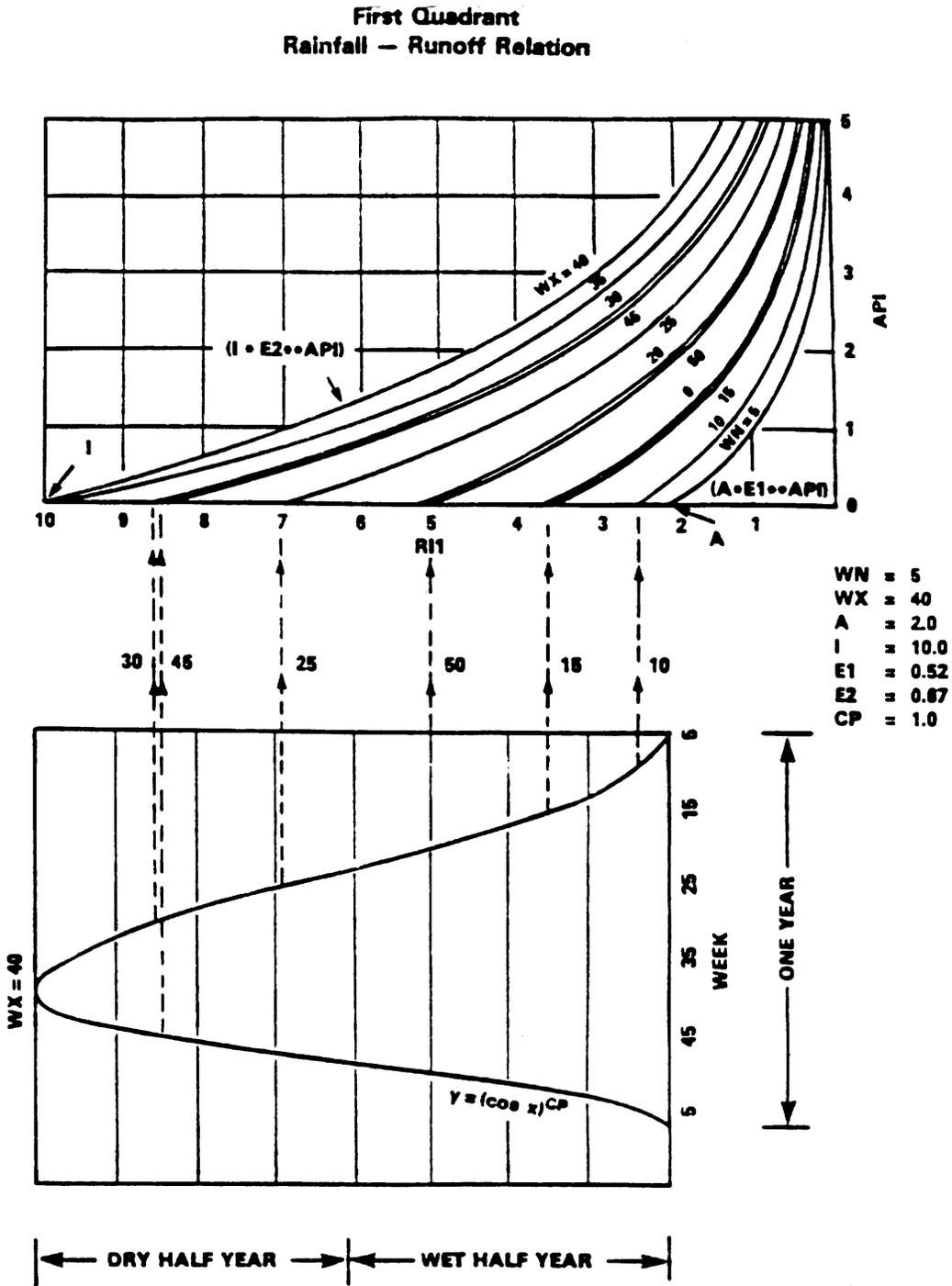


Figure 2. Modified API -- Salt Lake City RFC

Figure 3. First quadrant (seasonal relation)



The Cosine Function Determines the Intercept of Each Week with the ABSCISSA of the First Quadrant Curve (API = 0)

Figure 3. First quadrant (seasonal relation)⁷

Figure 4. First quadrant intercept function

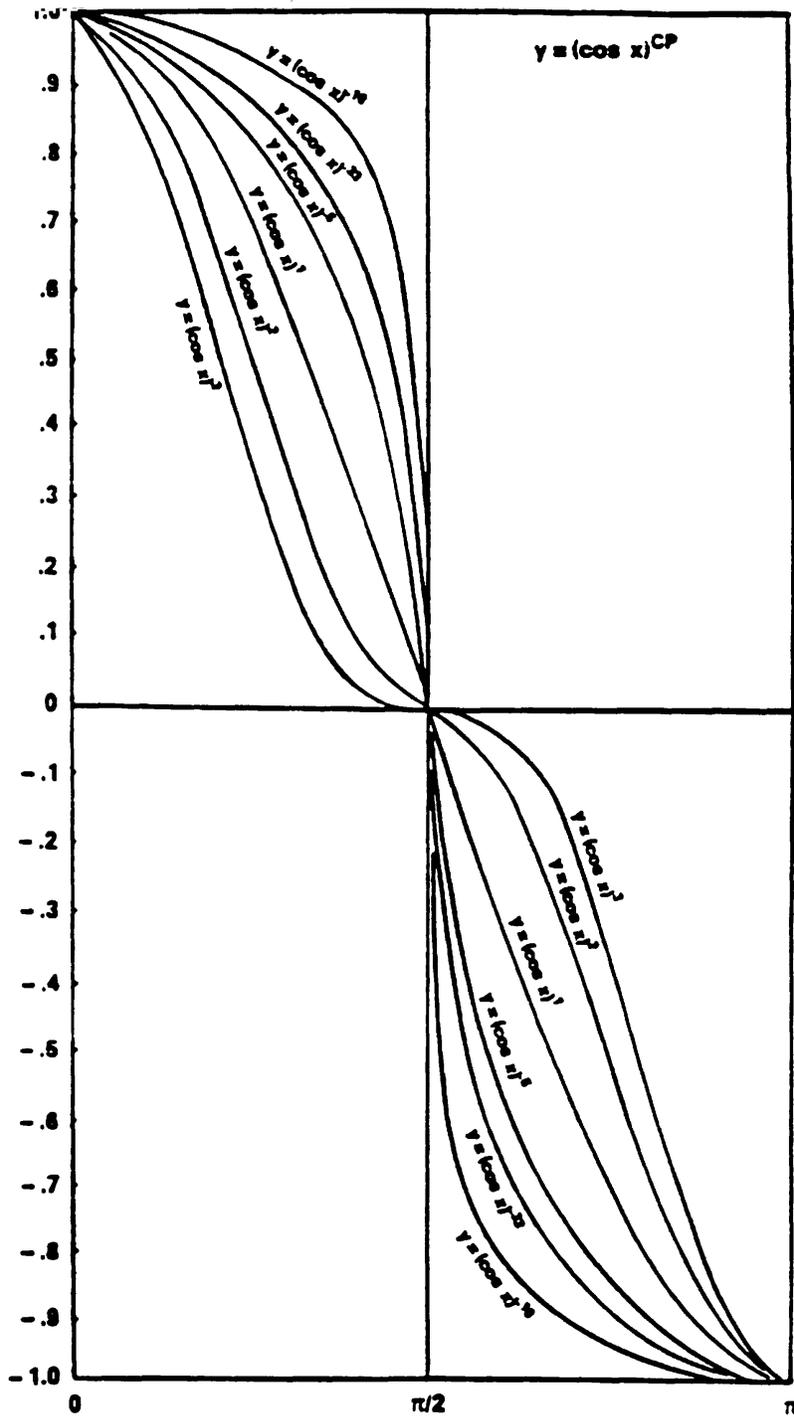


Figure 4. First quadrant intercept function⁷

Figure 5. API relation

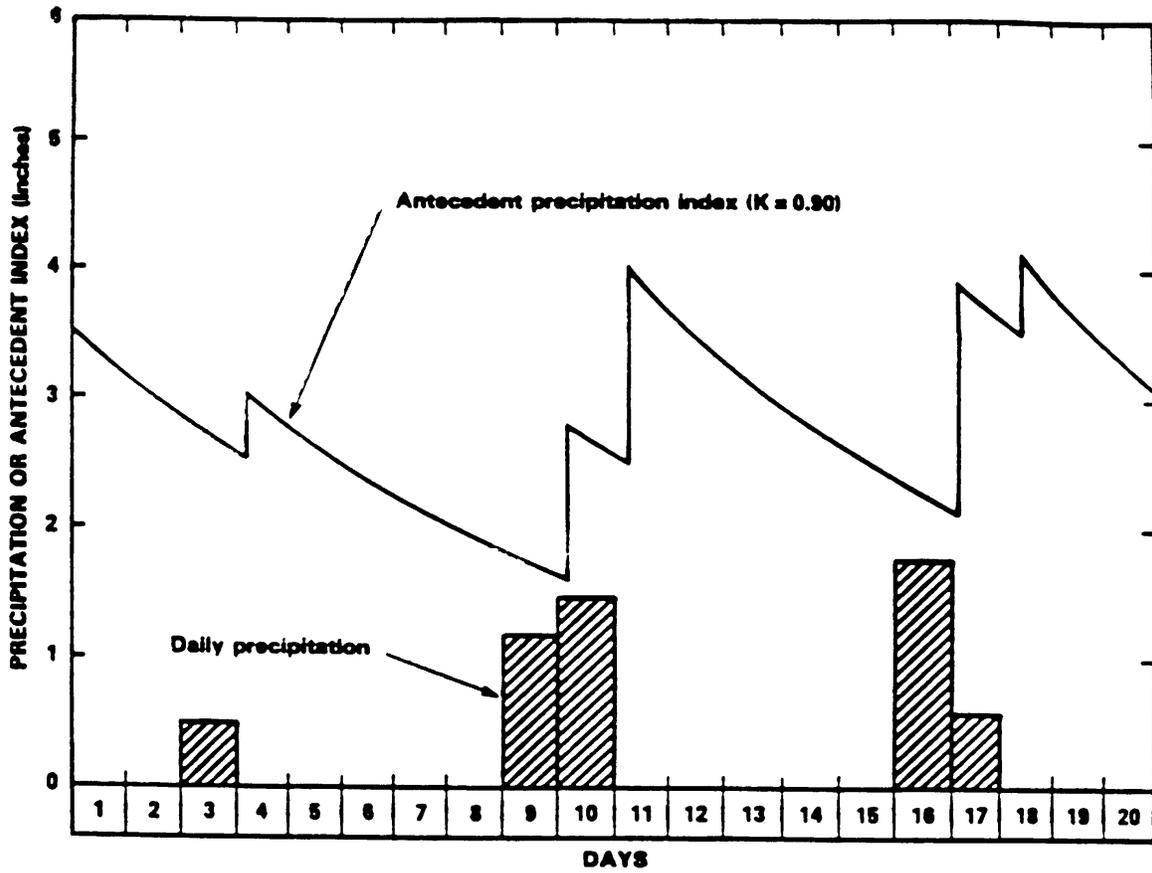


Figure 5. API relation